

# Higher Order Collins Modulations in Transversely Polarized Quark Fragmentation

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The Collins effect describes the modulation of the hadron production by a transversely polarized quark with the sine of the polar angle,  $\varphi$ , between the produced hadron's transverse momentum and the quark spin. We employ a quark-jet model to describe multiple hadron emissions by such a quark, taking the Collins effect into account. The resulting hadron distributions exhibit modulation up to fourth order in  $\sin(\varphi)$  when only two hadron emissions are allowed, rising with any further increase in the number of emitted hadrons. These new effects are a direct consequence of the quark-jet mechanism for quark hadronization, which do not depend on the details of the model used for elementary hadron emission. The size and the sign of the higher order terms are directly connected with the probabilities of quark spin flip in the elementary emission process, with opposing sign favored and unfavored Collins functions only being generated if quark spin flip is preferential. Experimental studies of these effects should therefore provide a critical test of the quark hadronization mechanism, which in turn will lead to a deeper understanding of the transverse spin structure of hadrons.

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Quark hadronization remains one of the most challenging problems in hadronic physics, as it involves both long and short-range parton interactions. This process is quantitatively described by various fragmentation functions, which have a probabilistic interpretation. For example, the unpolarized fragmentation function,  $D_1^{h/q}(z)$ , can be interpreted as the probability density for an unpolarized quark  $q$  to emit a hadron  $h$  carrying light-cone momentum fraction  $z$ . These functions enter the expressions for the cross-sections of various hadronic processes in the factorization regime, such as semi-inclusive deep inelastic scattering (SIDIS) and  $e^+e^-$  and hadron-hadron collisions, convoluted where necessary with the relevant parton distribution functions. Thus, a reliable knowledge of the fragmentation functions is crucial for disentangling the underlying parton distribution functions.

In recent years there has been particular interest in studying scattering processes with transversely polarized probes and targets. Here the fragmentation of the transversely polarized quarks to spin zero particles, like pions and kaons, can be described by two functions: the familiar unpolarized fragmentation function and the naïvely time-reversal-odd Collins function [1, 2].

The probability of the transversely polarized quark  $q$  to emit a spin-zero hadron  $h$  with transverse momentum  $\vec{P}_\perp$ , depicted schematically in Fig. 1, can be expressed as [3]

$$D_{h/q\uparrow}(z, P_\perp^2, \varphi) = D_1^{h/q}(z, P_\perp^2) - H_1^{\perp h/q}(z, P_\perp^2) \frac{P_\perp S_q}{zm_h} \sin(\varphi), \quad (1)$$

where  $m_h$  is the mass of the produced hadron. The trans-

verse momentum dependent (TMD) unpolarized fragmentation function is denoted by  $D_1^{h/q}(z, P_\perp^2)$ , while  $H_1^{\perp h/q}(z, P_\perp^2)$  is the Collins function. Thus the Collins function modulates the distribution of produced hadrons with  $\sin(\varphi)$ , a clear signature that allows one to extract it from the experimental data. The experimentally measured Collins functions from HERMES, COMPASS and JLab strongly suggest that the 1/2 moments

$$H_{1(h/q)}^{\perp(1/2)}(z) \equiv \pi \int_0^\infty dP_\perp^2 \frac{P_\perp}{2zm_h} H_1^{\perp h/q}(z, P_\perp^2) \quad (2)$$

of the unfavored Collins functions have a similar size and opposite sign to those for the favored ones for pions [4–8]. We note that the fragmentation functions are called favored, where the produced hadron has a valence quark of the same flavor as the initial fragmenting quark, and unfavored otherwise. There has been extensive modeling of the Collins function in the past [9–12]. Most notably, the spectator model calculations of Refs. [9–11] provide a microscopic description of the fragmentation process. However, these calculations have been limited to the single hadron emission approximation, with the consequence that the unfavored fragmentation functions are zero in this approach. In addition, the description of the small  $z$  region for the favored fragmentation functions requires the introduction of a phenomenological quark-meson form factor.

Recently, we employed the NJL-jet model of Refs. [13–16] to calculate the Collins fragmentation function within the quark-jet hadronization picture [17], using Monte Carlo (MC) simulations. In this picture, the initial quark undergoes a decay chain of hadron emissions, schemati-

cally depicted in Fig. 2, where a microscopic description of each elementary emission is provided by the effective quark model of Nambu and Jona-Lasinio (NJL) [18].

In this Letter we show that within this picture the fragmentation function of a transversely polarized quark acquires Collins modulations of higher order in  $\sin(\varphi)$  as a direct consequence of the quark-jet hadronization process. These new terms are essential to describe the unfavored fragmentation functions. It appears that, even with just two hadron emissions in the decay chain, one requires a fourth order polynomial in  $\sin(\varphi)$  to fully describe the produced number densities. We stress that this effect is independent of the particular form of the model used to describe the Collins effect at the elementary emission vertex but is a direct consequence of the multiple hadron emission mechanism.

In the NJL-jet model of Matevosyan *et al.* [17] we used the tree level diagrams to calculate the unpolarized fragmentation functions (the first term in Eq. (1)), while for the Collins functions we used the spectator model of Bacchetta *et al.* [10]. In that work, the interference term between a tree level amplitude and the amplitude with gauge link coupling was used to produce non-zero Collins function.

A key element of this calculation was the evaluation of the quark spin-flip probability in each hadron emission step. We used light-cone spinors to describe the initial and final polarized quark wave-functions, and found that the quark spin non-flip and spin flip probabilities are proportional to

$$|a_1|^2 \sim l_x^2, \quad |a_2|^2 \sim l_y^2 + (M_2 - (1-z)M_1)^2, \quad (3)$$

where  $M_{1(2)}$  is the constituent mass of the fragmenting (remnant) quark and  $l_\perp$  is the transverse momentum of the remnant quark. (Note that this momentum cancels

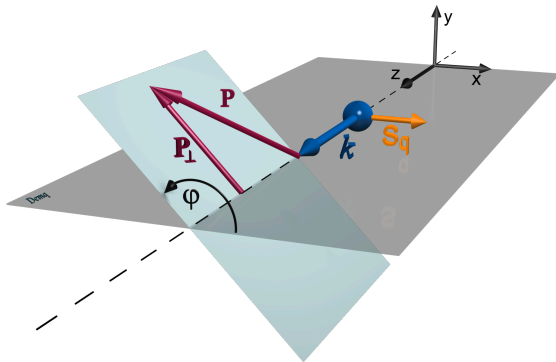


FIG. 1. Illustration of transversely polarized quark fragmentation. The quark momentum,  $\mathbf{k}$ , defines the  $z$ -axis, with its transverse polarization spin vector,  $\mathbf{S}_q$ , along the  $x$  axis. The emitted hadron has momentum  $\mathbf{P}$  with component  $\mathbf{P}_\perp$  transverse with respect to the  $z$ -axis. The polar angle of  $\mathbf{P}_\perp$  with respect to  $\mathbf{S}_q$  is denoted  $\varphi$ .

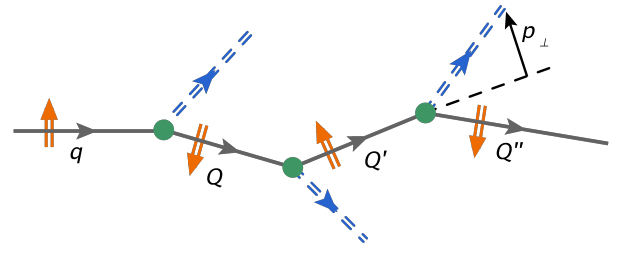


FIG. 2. NJL-jet model including transverse momentum and quark polarization transfer. Here the (orange) double-lined arrows indicate the quark's spin.

the transverse momentum of the emitted hadron with respect to the momentum of the fragmenting quark.) Here we clearly see that the quark spin-flip probability should be on average larger than the spin non-flip probability. This will be shown later to be crucial in describing the Collins function.

We use the elementary fragmentation function for a transversely polarized quark calculated within the NJL model, along with the quark spin flip probabilities explained above, as the input to a Monte Carlo simulation of the quark hadronization process within the quark-jet model – see Ref. [17] for a more detailed discussion. We used the  $z$ ,  $P_\perp^2$  and  $\varphi$  dependence of the resulting hadron number densities to extract the unpolarized and Collins functions for pions and kaons produced by light and strange quarks. The transverse momentum integrated number densities were perfectly described by a first order polynomial in  $\sin(\varphi)$  and hence the constant term was interpreted as the integrated unpolarized fragmentation function, while the coefficient of the linear term as twice the  $1/2$  moment of the Collins function (c.f. Eq. (2)). As perhaps the most remarkable result, we note that the unfavored Collins function was found to have the opposite sign and a magnitude comparable to the favored one for pions, as can be seen from the plot in Fig. 3.

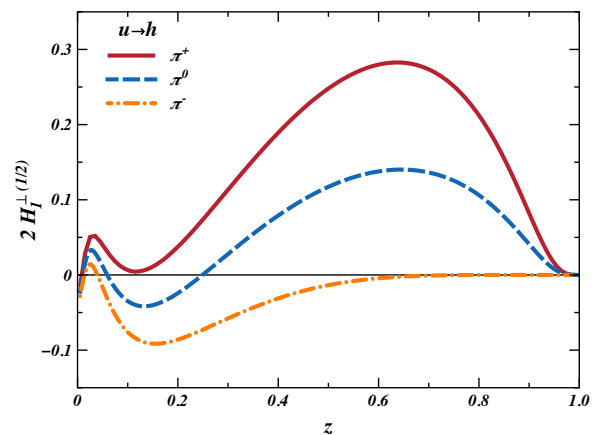


FIG. 3. Fitted values for  $2H_1^{\perp(1/2)}$  for pions produced by a  $u$  quark from MC simulations with 6 emitted hadrons.

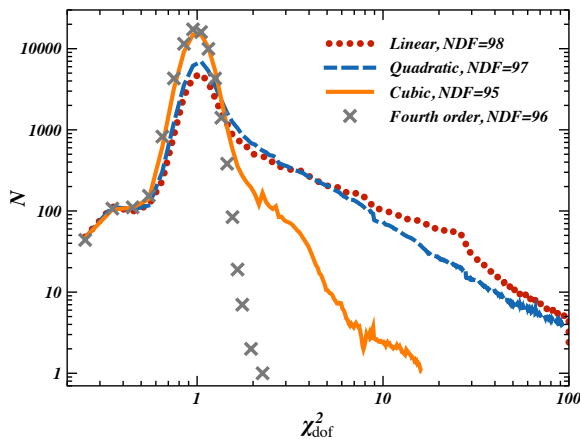


FIG. 4. Histogram of the values of  $\chi^2_{\text{dof}}$  for all fragmentation functions of  $u$  quark fitted with polynomials of different order in  $\sin(\varphi)$  for MC simulations with 2 emitted hadrons.

Even more interesting effects were observed when we considered the extraction of the TMD Collins function from the unintegrated number densities. We used a polynomial form in  $\sin(\varphi)$  to minimise  $\chi^2$  for a fit to the  $\varphi$  dependence for fixed values of  $z$  and  $P_\perp^2$ . Here, even for only two hadron emissions in the jet, the fits with just a linear form were insufficient. On the other hand, a fourth order polynomial in  $\sin(\varphi)$  provides an excellent fit everywhere. The plots in Fig. 4 illustrate the  $\chi^2$  per degree of freedom ( $\chi^2_{\text{dof}}$ ) for fits with polynomials of different orders for all the fragmentation functions of  $u$  quark (to seven pions and kaons in total) and for all the discrete values of  $z$  and  $P_\perp^2$  in our simulations – a total of  $7 \times 100 \times 100 = 7 \cdot 10^4$  data points. Clearly the fourth order polynomial yields  $\chi^2_{\text{dof}}$  in the vicinity of 1 for all the fits. A similar analysis for the  $P_\perp^2$  integrated number densities shows that a simple first order polynomial describes all the data perfectly well, meaning that this effect is only present in the TMD fragmentations.

These higher order modulations can be attributed to two effects. First, in the process of multiple hadron emissions, the distribution of the  $n$ -th produced hadron is multiplied by an additional factor of  $(a + b \sin(\varphi))^{(n-1)}$ , when averaged over all Monte Carlo simulations. This can be easily deduced from Eq. (1), as with each hadron emission the remnant quark number densities, relative to the fragmenting quark ( $d_q^Q$ ), are related to the number density of the emitted hadron ( $d_q^h$ ) by the momentum and flavor conservation law  $d_q^Q(1 - z, -\mathbf{p}_\perp) \propto d_q^h(z, \mathbf{p}_\perp)$ , where  $\mathbf{p}_\perp$  is the transverse momentum of the produced hadron with respect to the fragmenting quark. Thus these quark densities also get modulated with a first order polynomial form. The next emitted hadron's distribution is modulated not only by the elementary Collins function, but also by the number density of the fragmenting quark, yielding a quadratic  $\sin(\varphi)$  term in the angular depen-

dence of not only the produced hadron but also the next remnant quark in the decay chain. Consequently, from this effect alone the distribution of each consecutive emitted hadron acquires another order in  $\sin(\varphi)$ . For a detailed investigation of this effect we refer to Matevosyan *et al.* [19], who employed a toy model to study the effect of quark spin flip in the decay chain for unfavored fragmentation functions.

The second source of the angular modulation arises from the spin flip probabilities themselves. This can be easily seen from the expressions for the spin flip probability of Eq. (3), where  $\vec{l}_\perp = -\vec{p}_\perp$  and  $p_x = p_\perp \cos(\varphi)$ ,  $p_y = p_\perp \sin(\varphi)$ . Thus both  $|a_1|^2$  and  $|a_2|^2$  are modulated with a polynomial of the form  $a + b \sin^2(\varphi)$ . Together these effects create an angular modulation of the hadrons produced at  $n$ 'th order that is a polynomial in  $\sin(\varphi)$  of order  $1 + 3 \times (n - 1)$ . Then we can write the general form for the polarized fragmentation function for  $R$  hadron emissions as

$$D_{h/q^+}(z, P_\perp^2, \varphi) = \sum_{n=0}^{1+3(R-1)} c_n(z, P_\perp^2) \sin^n \varphi. \quad (4)$$

Thus, in the case with only two hadron emissions, we need a fourth order polynomial to fully describe the angular modulation, as was shown in our  $\chi^2$  fits in Fig. 4.

Figure 5 shows the coefficients,  $c_n$ , of the  $n$ 'th power of  $\sin(\varphi)$ , obtained from a  $\chi^2$  fit to the fragmentation function for a  $u$  quark going to a  $\pi^+$  at  $z = 0.1$  and to a  $\pi^-$  at  $z = 0.3$ , as a function of  $P_\perp^2$ . In this case the Monte Carlo simulations involved six emitted hadrons (beyond which the results are stable). Here we do not show the constant term corresponding to the unpolarized fragmentation function, as it is much larger and has been extensively studied in our previous work [16, 17]. The linear term,  $c_1$ , corresponds to the Collins function. The plots clearly show that the higher order terms follow the pattern set by the linear Collins term studied in Ref. [17]: they vanish for large values of  $P_\perp^2$  and oscillate with an increasing magnitude with  $P_\perp^2 \rightarrow 0$ . In addition, the higher order terms are comparable in magnitude to the Collins terms in the small  $z$  region. It is worth noting, that the higher order terms for unfavored fragmentation remain significant compared to the regular Collins term to substantially larger values of  $z$  than the favored fragmentations.

We note that the hadrons produced further down the quark decay chain carry increasingly smaller momentum fractions and thus impact the distribution functions only in the low  $z$  region. This was shown explicitly for the integrated, unpolarized fragmentation function in Ref. [15], where the plots in Fig. 5 demonstrate that the fragmentation functions hardly change after just three hadron emissions in the region  $z \gtrsim 0.2$ . Thus the higher order modulations of the distributions appear mainly in the low  $z$  region, making it harder to isolate the higher order

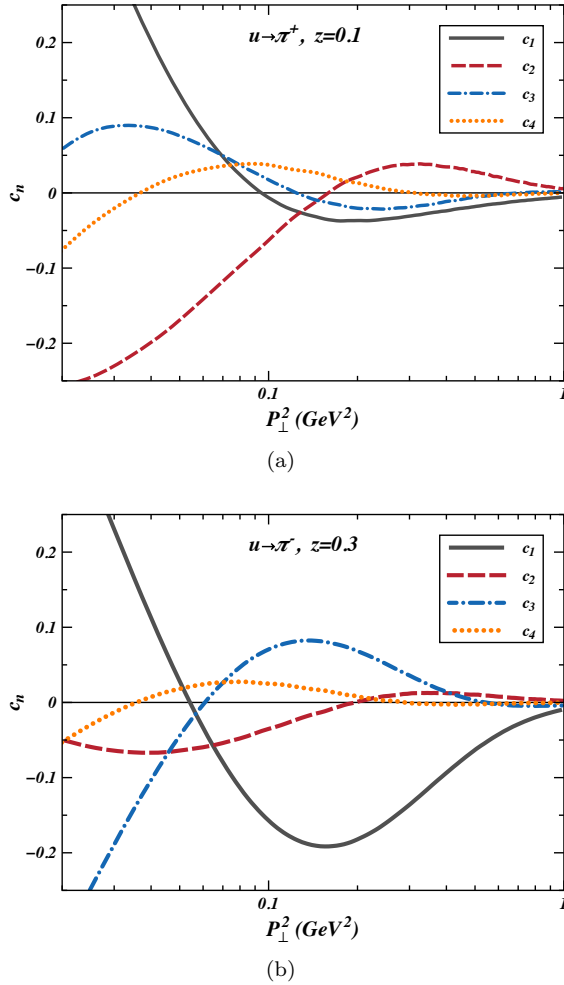


FIG. 5. The fitted values for the polynomial coefficients in  $\sin(\varphi)$ ,  $c_n$ , (a) for  $\pi^+$  at  $z = 0.1$  and (b) for  $\pi^-$  at  $z = 0.3$ , produced by a  $u$  quark for simulations with 6 emitted hadrons.

polynomial terms in the MC data. Thus, a fourth order polynomial was sufficient to describe our MC results within the statistical errors with 6 hadron emissions.

In this work we have described for the first time the higher order Collins modulation effects for the hadrons produced in the fragmentation of a transversely polarized quark within the quark-jet hadronization picture. We showed that these higher order polynomial terms in  $\sin(\varphi)$  necessarily arise from two effects in each hadron emission step: the linear Collins modulation of the remnant quark number densities in the decay chain and the  $\sin^2(\varphi)$  modulation of the quark spin flip probabilities. In principle, this leads to an increase of the angular modulations of the resulting hadron distributions by three orders in  $\sin(\varphi)$  with each hadron emission after the first one. We showed that with just a linear Collins modulation as an input in the elementary quark emission our Monte Carlo results require a fit with a polynomial of at least fourth order to achieve satisfactory values of  $\chi^2_{\text{dof}}$ . Earlier toy model tests in Ref. [19] helped to establish this mech-

anism of generating higher order modulations. Moreover, they showed that the experimental suggestion that the  $1/2$  moments of the favored and unfavored Collins functions may have opposite signs, may be understood in the quark-jet picture if, on average, in the elementary hadron emission step the remnant quark's spin is antiparallel to that of the fragmenting quark.

These new Collins modulation effects are only visible in the transverse momentum dependent hadron number densities and grow relative to the unpolarized and regular Collins terms in the small  $z$  and  $P_\perp^2$  region, where the multiple hadron emission effects are the largest. Nevertheless, if observed experimentally, these effects will provide a critical test of the quark hadronization process. Further, their inclusion in the analysis of high precision experimental data will allow us to improve the description of the data and reduce the systematic errors in the extractions of the unpolarized and Collins fragmentation functions.

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